A Note on Differential Corner Measures

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Abstract

Corner detection plays a central role in many image analysis applications ranging from character recognition to landmark identification. We study the isophote curvature times the gradient magnitude to some power as measures of cornerness and we focus on tracking of extremal points of these measures across scales. We conclude that annihilations and creations generically occurs, implying that corners in general cannot be tracked to arbitrarily fine (or coarse) scale. The analysis indicates that isophote curvature times the gradient magnitude is best suited for binary images.

Keywords: Corners, Catastrophe Theory, Singularity Theory, Isophote Curvature, Gradient Magnitude, Linear Scale-Space, and Character Analysis.

1 Introduction

Corner detection plays a central role in many image analysis applications ranging from character recognition to landmark identification. The literature on corner detection roughly divides into two classes. Some use explicit models, see e.g. [11] for an overview. Others use derivative expressions like the Gaussian curvature, the structure tensor (interest operator, second moment matrix), expressions involving the isophote curvature, and the curvature of Canny edges, see e.g. [12] for an overview.

One subclass of the latter is corners defined as the isophote curvature times the absolute gradient length to some power a:

$$C = |\nabla L|^a \kappa = L_w^{a-1} L_{vv} \tag{1}$$

where w is the gradient direction and v is the (perpendicular) tangent direction of the isophote in a right hand coordinate system (w, v). Kitchen and Rosenfeld [5] suggested to use $a = 1$, Zuniga and Haralick [15] proposed $a = 0$, and Blom [1] and Lindeberg [6] investigated $a = 3$.

The advantage of using a corner measure with $a > 0$ is that the product will focus on high isophote curvatures close to high contrast edges. There are two special values of a that deserve a note: $a = 0$ is invariant under monotonic transformation of the image intensities (morphological invariance), and $a = 3$ is invariant under affine transformations (the angle of the corner).

We will investigate the above subclass of corner measures in an embedded Gaussian Scale-Space (see [14] and the references therein):

$$L(x, t) = G(x, t) * L(x)$$

where the original image $L(x)$ is convolved with a Gaussian G of variance $2t$.

The advantage of such an embedding is that it reduces the grid and noise effects and allows for a uniform analysis of corners of all sizes or resolutions. The disadvantage is that the corners are dislocated at high scale and should be traced back to low scale in order to improve their location. We will show that this process – although common in the literature – is problematic due to the complicated catastrophe structure across scale.

We will sketch the catastrophe structure in two different settings. Firstly, by examining the spatial singularity structure of the corner measures. However, Rieger [9] noted that such corner points usually do not correspond to corners of Canny edges. Therefore in a second approach we will extend Rieger's analysis of corners on Canny edges to edges defined as single isophotes.

Related to this work in terms of Catastrophe Theory is Damon [2], Rieger [9, 10], Griffin & Colchester [3], Olsen [7], and Johansen [4].

2 Image structure

Assume a multi-scale 2D image $L(x, y, t) : \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{R}$, where x, y are the spatial coordinates and t a scale parameter. We are interested in spatial point features defined as the intersection of zero loci of two differential expressions: $A(x, y, t) = 0$ and $B(x, y, t) = 0$. In our case of corners, this may be $A = \partial_x C$ and $B = \partial_y C$, where $C = |\nabla L|^a \kappa$, $a \in [0; 3]$, or in the case of corners constrained to a single isophote: $A = L - I_0$ and $B = \partial_v C$, where I_0 is a given isophote value. We analyse the scale-space curves satisfying $A = B = 0$. In the scale-space points where the tangent of the curve is not pointing directly in the scale direction, we may introduce a local parametrisation of the curve

$$A(x, y(x), t(x)) = 0, \; B(x, y(x), t(x)) = 0$$

such that it is identified by the two scalar functions $y(x)$ and $t(x)$. By differentiation with respect to x and solving a linear system of equations, we obtain:

$$t_x = \frac{A_x B_y - B_x A_y}{A_t B_y - A_y B_t}$$

The denominator is only zero when the tangent of the curve points in the scale direction, and we find that the curve is horizontal only when the numerator is zero. These horizontal points corresponds generically to two curves that meet at one scale yielding an annihilation or creation of a pair of feature points.

Whether it is an annihilation or creation for increasing scale can be accessed through the sign of t_{xx}: negative for annihilation and positive for creation. If the second order structure t_{xx} vanishes, we get an event of even higher order.

The Gaussian scale space image satisfies the heat equation ($\partial_t = \sum_i \partial_{x_i x_i}$), changing the general program of catastrophe theory slightly [2]. To describe the local jet in space and scale we develop the image in heat polynomials, i.e. polynomials satisfying the heat equation. In 1D they can be generated by the following recursion formula

$$v_n = x v_{n-1} + 2(n-1) t v_{n-2}, \quad v_0 = 1, \quad v_1 = x.$$

A local polynomial model of the 2D image may then be constructed from the jet (the local derivative structure) as

$$\tilde{L} = a_{00} v_0 + a_{10} v_1(x) + a_{01} v_1(y) + a_{20} v_2(x) + a_{11} v_1(x) v_1(y) + \ldots,$$

where the a's are constants proportional to the spatial derivatives in $(x, y, t) = (0, 0, 0)$ by factorial factors.

Given $\tilde{L}$, we compute $\tilde{A}, \tilde{B}$. We count the linear constraints on the jet (the a's) to be satisfied for a given event to happen. In general we have 3 translational degrees of freedom of the coordinate system so that in a generic image we can satisfy linear constraints on three different coefficients. Furthermore we can choose the spatial rotation of our coordinate system freely to simplify expressions.

We have analysed the corner definitions outlined above. In all cases, the curves have generically points at which the curve's tangent has no scale component and second order curve structure such that *both creation and annihilation will generically happen.* On top of this, the case $a = 3$ has higher order events happening in critical points of the image, but they are of no interest when considering maxima of $|C|$ since $C = 0$ here. The events for critical points in C are the approach of a saddle to a minimum or maximum. The events for the isophote constrained measure is that a minimum on the curve meets a maximum. In both cases, there is no constraint on whether minima must be positive or maxima negative or vice versa. Hence we can conclude that both annihilation and creation happens generically involving maxima of the absolute value of C. The implication of this is that *tracking a corner over scale can only be performed over an open interval of scales.* At scales outside this interval, the corner does not exist. In Figure 1–3, we have shown the critical points of C on the letter 'c' over scale for the four different corner measures.

3 Experiments on Characters

The experiments we have performed are on binary images of characters. The quantisation implies that the images are non-generic at lowest scale but will behave as a generic image, when the scale is increased. This further implies that edges ($L_{ww} = 0$) initially will be close to the mid-isophote (midway between light and dark), and we may hence approximate their behaviour as the behaviour of the mid-isophote. This also suggests that the image will evolve initially according to Eucledian Shortening Flow, since the isophotes evolve according to $\partial_t S = \left(\kappa + \frac{L_{ww}}{L_w}\right)\vec{N}$ in Linear Scale-Space, where S is an isophote, κ is its curvature, and $\vec{N}$ is its normal [8]. We will thus expect creations to be high scale phenomena.

In Figure 1–4 some experiments on the letter 'c' are shown. From these experiments we observe that both the spatial extremal and the single isophote approaches display similar behaviour w.r.t. the following points: Creation events occur, localisation is poor at high scale, and finally, the number and localisation of critical points at low scale is similar for all a, but the evolution is very different.

Conversely, the spatial extremal approach is very sensitive to noise with respect to the topology (notice the shear explosion in critical points in bottom box of Figure 1 in comparison with the top box). The single isophote approach is so stable that we have chosen not to display the noisy version of Figure 2; there was no visual difference.

Finally, we conclude by Figure 4 that when the corners of a single isophote are ordered according to their absolute strength, varying a changes this ordering. Notice especially that the peak at approx. arc-length 40 is practically removed when a is increased while its neighbour at approx. arc-length 20 becomes the dominating corner point for $a = 2$.

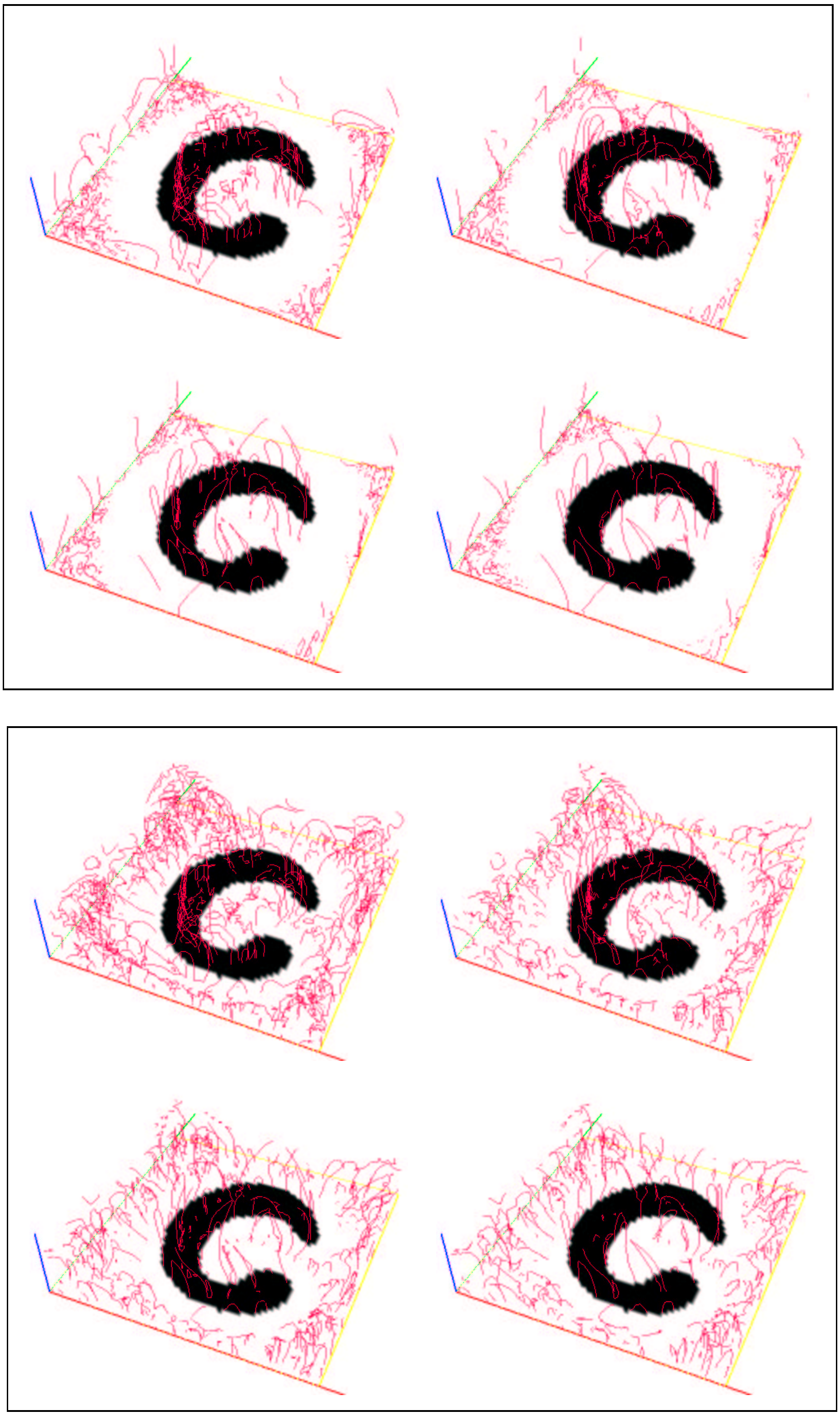

Figure 1: Two experiments evaluating the critical points of the image of 'C' for $a = 0, \ldots, 3$ reading from left to right and top down. Top box is for a noiseless images while the image in the bottom box has artificially been corrupted by the addition of normal distributed noise of mean 0 and standard deviation 5.

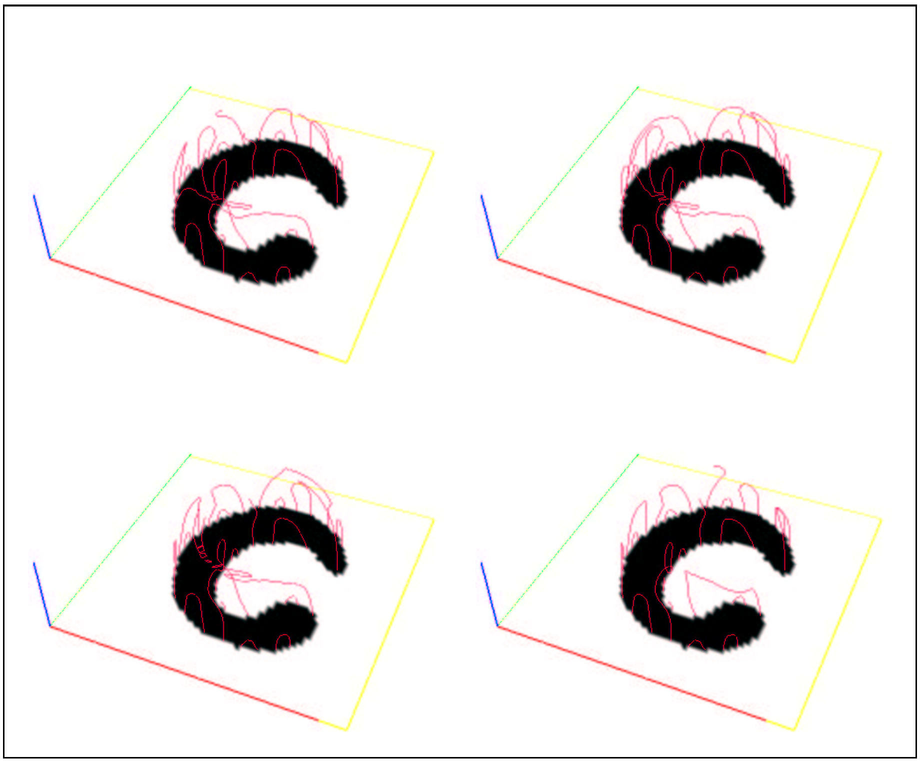

Figure 2: The critical points of the mid-isophote of 'C' for $a = 0, \ldots, 3$ reading from left to right and top down. A zoom is given in Figure 3.

4 Summary

We have studied the family of corner measures $\kappa|\nabla L|^a$ for $a \in \{0, \ldots, 3\}$ embedded in Gaussian Scale-Space. Two approaches have been used: The catastrophe structure of both the spatial extrema of the corner measure and the extrema along an isophote.

For both approaches we have concluded that the value $a = 1$ seems to be the simplest with respect to the number of catastrophes. Especially for $a = 0$ many catastrophes appear.

Finally, the isophote approach has been shown to shift the focus away from high isophote curvature points for $a > 1$. The resulting corners do not correspond well with intuition.

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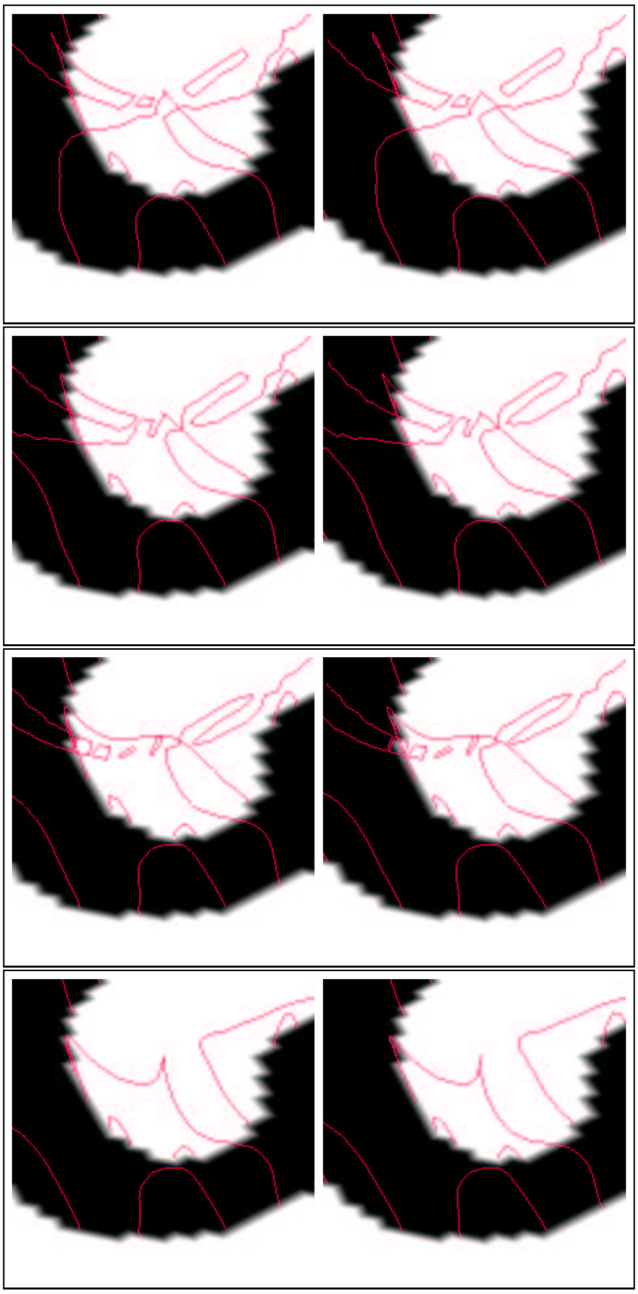

Figure 3: Stereo pairs showing a zoom of the lower part of the 'C' from Figure 2 for $a = 0, \ldots, 3$ reading from top to bottom. Note the significant difference in the catastrophes

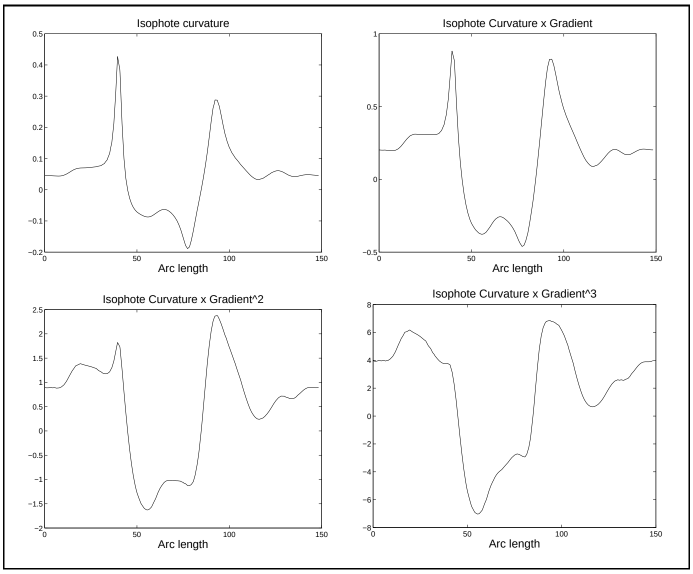

Figure 4: $L_w^{a-1}L_{vv}$ for the letter c at $t = 61.8$ and for $a = 0, \ldots, 3$ reading from left to right and top down. For the isophote curvature ($a = 0$) the arc-length functions begins on the outer side of 'c', reaches the sharpest corner corresponding to the maximal peak, travels along the inner side of 'c' yielding negative curvature and reaches the onset of the outer part again at approximately arc-length 90. The same arc-length function is used for the other corner measures as well.

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